MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

PSM0325 – INTRODUCTION TO PROBABILITY AND STATISTICS

(Foundation in Information Technology / Life Sciences)

7 MARCH 2019 2.30 p.m –4.30 p.m (2 Hours)

INSTRUCTIONS TO STUDENTS

- 1. This question paper consists of **THREE** pages excluding the cover page and the Appendix.
- 2. Answer ALL FIVE questions. All questions carry equal marks and the distribution of the marks for each question is given.
- 3. Please write all your answers in the Answer Booklet provided. All necessary working steps MUST be shown.
- 4. Statistical table is provided.

Instruction: Answer all FIVE questions.

Question 1 (10 marks)

a. The height, in cm, of 30 students who enrolled for the physical training is given below:

146	158	168
150	159	171
150	160	173
153	160	176
154	160	178
155	163	178
155	163	178
156	163	178
157	165	180
158	165	183

- i. Construct a frequency and cumulative frequency distribution such that lower limit of first class is 146 and the class width is 8. (4 marks)
- ii. Draw the frequency histogram.

(2 marks)

b. Given the frequency distribution in the table below:

Class Limit	Class	Frequency	Cumulative
	Boundary		Frequency
0 - 4		17	
5 - 9		41	
10 - 14		22	
15 - 19		11	
20 - 24		8	
25 - 29		1	

- i. Copy the table and fill up values for the class boundary and cumulative frequency columns. (2 marks)
- ii. Find the median.

(2 marks)

Question 2 (10 marks)

a. The continuous random variable x has probability density function

given by
$$f(x) = \begin{cases} k(x^2 - 2x + 2) & 0 < x \le 3 \\ 3k & 3 < x \le 4 \end{cases}$$
 where k is a constant.

Continued...

i. Show that $k = \frac{1}{9}$

(3 marks)

ii. Find the mean of X.

(3 marks)

b. An experiment is defined as given:

Flip a fair coin once.

If a head shows up, select 2 balls, one at a time without replacement from a box.

If a tail shows up, select one ball at random from the same box.

The box contains 2 blue balls and 1 red ball.

Draw the tree diagram and find the probability that one red ball is selected.

(4 marks)

Question 3 (10 marks)

- a. Patients arrive at a district hospital emergency department at random at a rate of 6 patients per hour.
 - i. Find the probability that at most 2 patients arriving at the hospital emergency department. (2 marks)
 - ii. Find the probability that, during any 90 minute period the number of patients arriving at the hospital emergency department is at least 10.

(3 marks)

- b. The MUDA Screen Cinemas Company chain has studied its movie customers to determine how much money they spend on foods and drinks. The study revealed that the spending distribution is approximately normally distributed with a mean of \$7.11 and a standard deviation of \$1.37.
 - i. What percentage of customers will spend less than \$6.00 on foods and drinks? (2 marks)
 - ii. What is the minimum amount spend on food and drinks for the top 13%?
 (3 marks)

Question 4 (10 marks)

a. The following data shows the age of all 5 children in the family.

28 26 40 35 26

i. List all the possible samples of size four and determine its sample mean.

(4 marks)

ii. Construct the sampling distribution for the frequency and the probability of the sample mean. (2 marks)

Continued...

b. A sample is selected with the given data:

8.28, 2.87, 9.38, 9.73, 2.12, 2.42, 5.67, 2.58, 9.99, 5.27

i. Find the mean of the sample.

(1 mark)

ii. Construct a 90% confidence interval for the population mean, assuming the population standard deviation is 2.39. (3 marks)

Question 5 (10 marks)

- a. Write the null and alternative hypothesis for the following and determine if it is a two-tailed, a left-tailed or a right-tailed test.
 - A certain factory consumes on average 1000 m³ of water per day. A random sample of 100 days was taken to test if the mean daily water intake remains 1000 m³ against the alternative that the mean water consumption has increased. (2 marks)
 - ii. A snack food company produces a 250 gram bag of potato chip. Although the actual net weights deviate slightly from 250 gram and vary from one bag to another, the company insists that the mean net weight of the bags be 250 gram. (2 marks)
- b. A private telephone company provides a long distance telephone service. According to the company's records, the average length of all long distance call in 2010 was 12.44 minutes. The company's management wanted to check if the mean length of current long distance calls is different from 12.44 minutes. A sample of 150 calls placed through this company produced a mean length of 13.71 minutes with a standard deviation of 2.65 minutes.
 - i. Set up the hypothesis test, H_0 and H_1 . (2 marks)
 - ii. At the .05 significance level, can we conclude that the mean length of all current long distance calls is different from 12.44 minutes? (4 marks)

End of Paper

APPENDIX

Formulae:

1.

	Ungrouped data	Grouped data
Mean:	$\bar{x} = \frac{\sum x}{n} \qquad \mu = \frac{\sum x}{N}$	$\overline{x} = \frac{\sum mf}{n} \qquad \mu = \frac{\sum mf}{N}$
Variance:	$s^{2} = \frac{\sum x^{2} - \frac{\left(\sum x\right)^{2}}{n}}{n-1}$ $\sigma^{2} = \frac{\sum x^{2} - \frac{\left(\sum x\right)^{2}}{N}}{N}$	$s^{2} = \frac{\sum m^{2} f - \frac{\left(\sum mf\right)^{2}}{n}}{n-1}$ $\sigma^{2} = \frac{\sum m^{2} f - \frac{\left(\sum mf\right)^{2}}{N}}{N}$
Median:		$L + \left[\frac{\left[\frac{\sum f + 1}{2} \right] - F_L}{f_m} \right]_W$
Mode:		$L + \left[\frac{f_m - f_B}{(f_m - f_B) + (f_m - f_A)} \right] w$

2

Variance
$Var(X) = E(X^2) - [E(X)]^2 \text{ where}$
$E(X^2) = \sum_{x} x^2 P(x)$
$Var(X) = E(X^2) - [E(X)]^2$ where
$E(X^2) = \int_0^\infty x^2 f(x) dx$
<i>-</i>

3.

	Formula	Mean	Standard Deviation
Binomial Probability	$P(x) = \binom{n}{x} p^x q^{n-x}$	$\mu = np$	$\sigma = \sqrt{npq}$
Poisson Probability	$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$	$\mu = \lambda$	$\sigma = \sqrt{\lambda}$

4. The z value for a value of x:
$$z = \frac{x - \mu}{\sigma}$$

5. The z value for a value of
$$\overline{x}$$
: $z = \frac{\overline{x} - \mu_{\overline{x}}}{\sigma_{\overline{x}}}$ where $\mu_{\overline{x}} = \mu$ and $\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$

6. Sampling error =
$$\bar{x} - \mu$$

Non-sampling error = incorrect \bar{x} - correct \bar{x}

7. Point estimate of
$$\mu = \overline{x}$$

Margin of error = $\pm 1.96\sigma_{\overline{x}} = \pm 1.96\frac{\sigma}{\sqrt{n}}$ or $= \pm 1.96s_{\overline{x}} = \pm 1.96\frac{s}{\sqrt{n}}$

8. The
$$(1-\alpha)100\%$$
 confidence interval for μ is

$$\overline{x} \pm z \sigma_{\overline{x}}$$
 is known $\overline{x} \pm z s_{\overline{x}}$ if σ is not known

where
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$
 & $s_{\bar{x}} = \frac{s}{\sqrt{n}}$